Problem Solving and Data Analysis

Using quantitative reasoning, students will fluently solve problems using percentages, proportional relationships, ratios, rates, and units; analyze and interpret distributions of data; use various representations of data to find relative frequency, probabilities, and conditional probabilities; fit models to data and compare linear and exponential growth; and calculate, compare, and interpret mean, median, range, and standard deviation, understand basic study design, and interpret margin of error, all from high school courses preparatory for the math aligned with college and career readiness expectations. This Math Section Content Domain represents ≈15%/ 5–7 questions

Skill/Knowledge Testing Points

- Ratios, rates, proportional relationships, and units
- Percentages
- One-variable data: distributions and measures of center and spread
- Two-variable data: models and scatterplots
- Probability and conditional probability
- Inference from sample statistics and margin of error
- Evaluating statistical claims: observational studies and experiments

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Part VI: Line Graph, Histogram and Bar Graph

Part VII: Designing and Interpreting Experiments and Studies.

- ## Percent and percent change

Percent increase or decrease =

$$\frac{(Amount\ of\ increase\ or\ decrease)}{Original\ Amount} x\ 100$$

- ## Percent of a Solution (Mixture)

The percent of a solution is expressed as the percentage of solute over the total amount of solution.

E.g. Twenty gallons of 20% Alcohol contains

4 gallons of alcohol and 16 gallons of water

Solute = Alcohol

Solvent = Water

Solution= Solute + Solvent

% of a Solution is

$$\frac{Solute}{Total\ amount\ of\ solution}x100$$

$$\frac{(4\ gallons\ of\ alcohol)}{(4\ gallons\ of\ alcohol+16\ gallons\ of\ water)} x100$$

$$= 20\%\ Alcohol\ Solution$$

EXAMPLE 1 ☆☆☆

How many gallons of water must be added to 40 gallons of a 10% alcohol solution to produce an 8% alcohol solution?

A) 5

B) 8

C) 10

D) 12

Solution

Water has 0% alcohol, water $= x$

$(0)(x) + (0.1)\ (40) = (0.08)\ (x +40)$

$$4 = 0.08x + 3.20$$

$$0.08x = 0.8$$

$$x = 10$$

EXAMPLE 2

Some Athletes like to dilute 100% juice drink with water to lessen the flavor intensity and reduce caloric intake. Anna, a personal trainer, is preparing several blends of varying juice concentration to see which ratio her fitness clubs clients prefer, she plans to make 240 ounces each of 80% juice, 60% juice, 50% juice, 40% juice, and 20% juice blends. If the 100% juice Anna plans to buy comes in 32 ounces bottles and partial bottles are not sold, how many bottles of 100% juice will Anna need to buy to make her blends?

A) 8

B) 18

C) 19

D) 60

EXAMPLE 3

Political canvassers polled voters in two locations on whether they viewed a particular candidate for Governor favorably. At the first location, they asked 125 people and of those, 22.4 % responded favorably. At the second location, 37.5% of 272 people responded favorably, what percent of all the people surveyed responded favorably?

A) 25.7%

B) 30.0%

C) 31.5%

D) 32.7%

Solution

$$\frac{(0.224 \text{ x } 125) + (0.375 \text{ x } 272)}{125 + 272} \text{x}100$$

$$= 32.74\% \quad \text{…….. Ans. D)}$$

Solution

$(0.8)(240)+(0.6)(240)+(0.5)(240)+(0.4)(240)+(0.2)(240) = 32x$

$(2.5)(240) = 32x$

$x = 18.75…………..$Ans. (C)

EXAMPLE 5 ⭐⭐

A general contractor is building an addition onto a home he budgets 20% for materials, 55 % for labor, 10 % for equipment rental, and the rest is his fee. If the estimate the contractor gives to the homeowners says he will spend $5,200 on material, then how much is his fee?

A) $2600

B) $3900

C) $5200

D) $6500

EXAMPLE 6 ⭐⭐

In 1950, scientist estimated a certain animal population in a particular geographical area to be 6,400. In 2,000 the population had risen to 7,200. If this animal population experiences the same percent increase over the next 50 years, what will the approximate population be?

A) 8000

B) 8100

C) 8400

D) 8600

Solution

$$contractors\ fee = 15\%\ of\ budget$$

Since Material   = 20% of budget

Contractor's fee $= \dfrac{15}{20}$ x 5200

$= \$3900$

Or

If 20% of the budget is $5,200

then 100% of the budget is $26,000

Contractor fee $= \dfrac{15}{100}$ x $26,000

$= \$3,900$

Solution

Percent increase or decrease =

$$\dfrac{(Amount\ of\ increase\ or\ decrease)}{Original\ Amount}\ \text{x } 100$$

$$= \dfrac{7200-6400}{6400}\ \text{X } 100$$

$= 12.5\%\ (^1/_8)$

$= \dfrac{9}{8} \times 7200$

$= 8100$

EXAMPLE 7

A store is running a sale, discounting merchandise by a specific percentage. Raphael purchases a coffee maker with an original price of $62.00. After the sale discount, he gets an additional 10% off for using his store credit card. He ends up paying $41.85, not including tax. What was the amount of the original sale discount?

A) 15%

B) 22.5%

C) 25%

D) 75%

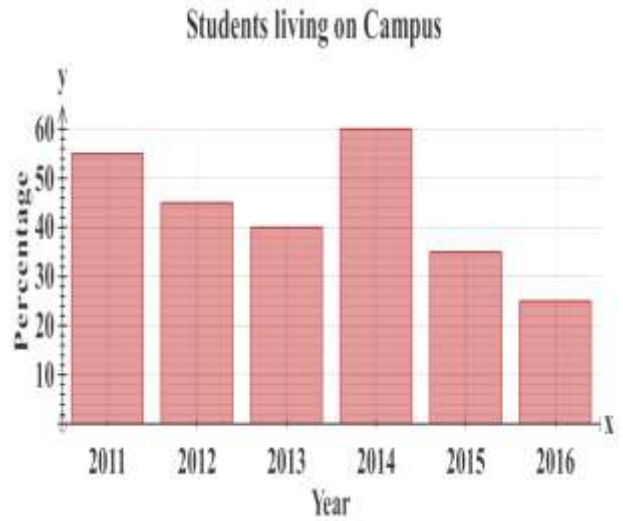**Solution**

$$(1 - x)(1 - 0.1)62 = 41.85$$

$$(1 - x) = \frac{41.85}{0.9 \times 62}$$

$$(1 - x) = 0.75$$

$$x = 0.25 \ or \ 25\%$$

……………………………..Ans. (C)

EXAMPLE 8

Students living on Campus

The graph above shows the percentage of students living on campus each year from 2011 to 2016 for a certain University.

In 2013 University meal plans were purchased by 80% of the students living on Campus and 25% of the students not living on Campus, What percentage of all the students at the University purchased a meal plan?
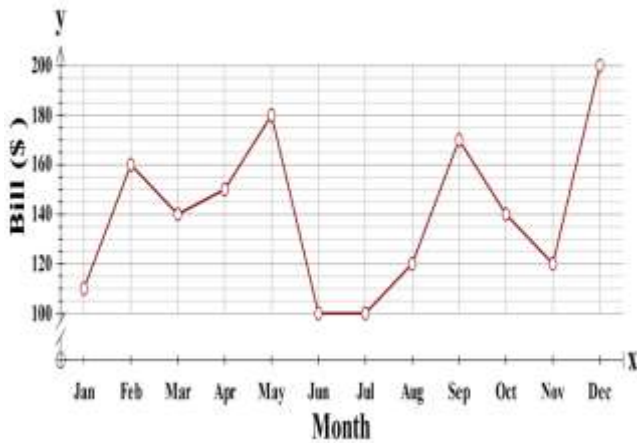
A)   20%

B)   25%

C)   47%

D)   75%

Solution

The graph shows 40 % of student lived on in 2013

Hence 60% lived off.

80% of 40% + 25% of 60%

= 32% +15%

=   47%   ………….Ans. (C).

EXAMPLE 9 ⭐⭐

Internet Bill

Month

This line graph above shows the monthly internet bill for a document center. What is the largest percentage change between consecutive monthly bills?

A) 44%

B) 45%

C) 67%

D) 75%

Solution

Look for the steepest lines, (May to June and Nov to Dec). The former is a percentage decrease and the latter is a percentage increase.

The percentage increase is always bigger with lines of similar steepness.

Percent increase or decrease =

$$\frac{(Amount\ of\ increase\ or\ decrease)}{Original\ Amount} \times 100$$

$$= \frac{(200-120)}{120} \times 100$$

$$= \quad \textbf{66.67\%} \quad ……….Ans. (C)$$

EXAMPLE 10 ⭐⭐⭐

The length of a rectangle is decreased by 25 percent and the width of the rectangle is increased by k percent. If the area of the rectangle increases by 5 percent, what is the value of k?

A)   25

B)   30

C)   35

D)   40

Solution

$$l \times w = A$$

$$(0.75)l \times \left(1 + \frac{k}{100}\right)w = (1.05)A$$

$$(0.75)\left(1 + \frac{k}{100}\right) = 1.05$$

$$1 + \frac{k}{100} = \frac{1.05}{0.75}$$

$$k = 40\ \% \quad …..Ans. (D)$$

EXAMPLE 11 ☆☆☆

One evening at a particular K.F.C. Restaurant, 68% of the customers ordered the Meal Deal, Which of the following could be the total number of customers in the K.F.C. Restaurant that evening?

A)   50

B)   55

C)   58

D)   60

Solution

$$68\% = \frac{17}{25}$$

The answer must be a multiple of the Denominator

…………………… Ans. (A)

EXAMPLE 12 ☆☆☆

Renata produces her own music and makes it available online to her subscribers. In November, she had 2.3 times as many subscribers as she had in May. What was the percent increase in her number of subscribers for this time period?

A) 2.3%

B) 23%

C) 123%

D) 130%

solution

$(2.3 - 1) \times 100 = 130\%$

…………………….Ans (D)

EXAMPLE 13 ☆☆☆

How many gallons of a 20% salt solution must be added to 10 gallons of 50% salt solution to produce 30% salt solution?

A)   5 gallons

B)   10 gallons

C)   15 gallons

D)   20 gallons

Solution

$$0.2x + 0.5(10) = 0.3(x + 10)$$

$$0.1x = 2$$

$$x = 20 \text{ …….. Ans. (D)}$$

1.

Industries in Canada May 2012

Goods-producing industry	Percent of GDP
Agriculture	5%
Construction	25%
Manufacturing	36%
Mining, oil, and gas extraction	26%
Utilities	8%

The total GDP for all five good-producing industries in Canada for May 2012 was approximately $474 billion. Which of the following is closest to the GDP for utilities, in the billions of dollars?

A) 8

B) 38

C) 59

D) 95

2.

The quantity k is reduced by 20% of its value. Which expression is equivalent to the resulting value?

A) $0.80k$

B) $0.20k$

C) $0.08k$

D) $0.02k$

3.

After a certain city passed an extra tax on bottles of water, the quantity of bottles of water purchased in 2012 decreased by 6.4% compared to 2011. If 10,000 bottles of water were purchased in 2011, how many bottles were purchased in 2012?

A) 10,640

B) 10,006

C) 9,994

D) 9,360